The OSA Fiber Modeling and Fabrication Technical Group Welcomes You

EVERYTHING YOU ALWAYS WANTED TO KNOW ABOUT SUPERCONTINUUM MODELLING IN OPTICAL FIBERS

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Current sample from another group
Technical Group at a Glance

• Focus
  • Development and application of high intensity lasers as well as novel XUV and x-ray sources
  • The physics of high intensity light interactions with matter
  • Short wavelength sources including insertion devices for storage rings (undulators and wigglers), plasma X-ray lasers, electron beam based sources and X-ray free electron lasers.

• Mission
  • To benefit YOU and to strengthen OUR community
  • Webinars, podcasts, publications, technical events, business events, outreach
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Today’s Webinar

Reaching for the brightest light at SLAC’s FACET-II

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Speaker’s Short Bio:
Everything you always wanted to know about supercontinuum modelling in optical fibers

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26.08.2019, OSA Webinar Series
What you will learn

• Concepts of Nonlinear fiber optics
  • the power of dispersion engineering

• A short overview of the most important **nonlinear effects** occurring during supercontinuum generation in optical fibers

• Describing the physical effects mathematically in the Generalized Nonlinear Schrödinger equation (GNLSE)

• **Solving the GNLSE** in an efficient way

• **Noise effects in supercontinuum generation** and how to include them into your simulation
Nonlinear optics – the tailor shop for light

Propagation of electromagnetic fields through fibers

linear
- Dispersion
- Absorption

nonlinear
- Four wave mixing
- Self phase modulation (SPM)
- Cross phase modulation (XPM)
- Modulation instability
- Raman scattering
- Brillouin scattering
- ...

intensity dependent!

Understand & Control Nonlinearity

Transform laser light to application demands
What intensities are we talking about?

“Everyday life intensities”
Intensity of sunlight on earth:
1000 W/m² = 0.1 W/cm²
When focused with a magnifying glass:
~ 100 W/cm² (max)

Intensities in nonlinear fiber optics:
Femtosecond laser pulse (100 fs, 10 nJ) focused into 2 μm core diam. fiber:
~ 1 000 000 000 000 W/cm²
= 1 TW/cm²

10 orders of magnitude higher than everyday life intensities!
Fibers for nonlinear optics

Revolution: photonic crystal fibers (PCF)

- Silica core
- Cladding of air-holes
- Design allows to “squeeze” the light into tiny cores (~ 1 μm)
  - very high intensities!
  - very high nonlinearity!
- Most important:

We can control nonlinear effects with the geometry of the fiber

This design flexibility only exists in fibers and waveguides!
Engineering the nonlinearity

Refractive index profile of specialty fibers

Single-mode fiber

Highly Nonlinear Fiber

Photonic Crystal Fiber

Taper / microfiber

Nonlinearity

large core
small index step
low confinement
low light intensity

(material)

small core
large index step
high confinement
high light intensity
Dispersion

- Refractive index of materials depends on the wavelength
- A laser pulse propagates in a medium with the group velocity

\[ v_g = c \left( n - \lambda \frac{dn}{d\lambda} \right)^{-1} \]

- The group velocity itself depends on the wavelength, i.e. there exists a Group Velocity Dispersion (GVD)

\[ D = -\frac{\lambda}{c} \frac{d^2n}{d\lambda^2} \]

D < 0: “Normal” dispersion: red faster than blue
D > 0: “Anomalous” dispersion: blue faster than red
Dispersion engineering

Most powerful tool in fiber optics

Dispersion is highly sensitive to the geometry of the air hole cladding

In pure silica: ZDW ≈ 1300 nm
Dispersion engineering

Most powerful tool in fiber optics

Dispersion is highly sensitive to the geometry of the air hole cladding

Control nonlinear effects
by tailoring the dispersion profile
by designing the geometry of the fiber
Nonlinear effects and dispersion
Most powerful tool in fiber optics

Occurrence of nonlinear effects

<table>
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<th>Anomalous dispersion</th>
<th>Normal dispersion</th>
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Control of nonlinear effects and their interaction by tailoring the dispersion!
Nonlinear effects in optical fibers

- Pulse propagation obeys

\[ \nabla \times \nabla \times E(r,t) + \frac{1}{c^2} \frac{\partial^2 E(r,t)}{\partial t^2} = -\mu_0 \frac{\partial^2 P(r,t)}{\partial t^2} \]

with

\[ P = \varepsilon_0 \left( \chi^{(1)} \cdot E + \chi^{(2)} : EE + \chi^{(3)} : EEE + \cdots \right) \]

Dispersion  Nonlinearities
Absorption

- \( \chi^{(2)} = 0 \) in silica! (centro-symmetric material)
  (no second-harmonic generation, sum frequency generation, etc)

- Only THIRD ORDER nonlinear effects in optical fibers!
Constructing a pulse propagation equation

Assumptions:

- Input pulse
  - linearly polarized along x-axis,
  - carrier frequency \( w_0 \)

- Fiber
  - single mode
  - polarization maintaining
  - propagation along z-axis

Ansatz:

\[
E(r, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{x}F(x, y, \omega)\tilde{A}(0, \omega)e^{i[\beta(\omega)z-\omega t]} \, d\omega
\]

- fiber mode profile
- spectral envelope
- phase shift
Constructing a pulse propagation equation

\[
E(r, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{x}F(x, y, \omega)\tilde{A}(0, \omega)e^{i[\beta(\omega)z-\omega t]} \, d\omega
\]

Some simplifications:

- **mode profile / effective mode area independent of frequency:**

  \[F(x, y, \omega) \rightarrow F(x, y, \omega_0)\]

  \[A_{\text{eff}}(\omega_0) = \frac{\left(\int_{-\infty}^{\infty} |F(x, y, \omega_0)|^2 \, dx \, dy\right)^2}{\int_{-\infty}^{\infty} |F(x, y, \omega)|^4 \, dx \, dy}\]

- **Taylor expansion** of propagation constant:

  \[\beta(\omega) = \beta_0 + (\omega - \omega_0)\beta_1 + \frac{1}{2}(\omega - \omega_0)^2\beta_2 + \cdots\]

  \[\beta_1 = v_g^{-1}, \quad \beta_2 > 0 : \text{normal GVD}, \quad \beta_2 < 0 : \text{anomalous GVD}\]

- **Transform to reference frame** co-moving with the input pulse

  \[T = t - \beta_1 z\]

  \[\Rightarrow \text{in practice we set} \quad \beta_0 = \beta_1 = 0\]
Constructing a pulse propagation equation

\[ E(r, T) = \hat{x} F(x, y, \omega_0) A(z, T) e^{i\omega_0 T} \]

\[ A(z, T) \]

A is normalized such that \( |A(z,T)|^2 \) yields instantaneous power in Watts.

Example for Gaussian input pulse:
\[ A(z = 0, T) = \sqrt{P_0} e^{-\left(\frac{T^2}{T_0^2}\right)} \]

Now: plug ansatz into Maxwell’s equations to derive equation for \( A(z,T) \)

\[ \Rightarrow \] details: e.g. Agrawal, Nonlinear Fiber Optics (Academic Press, 2013)
Generalized Nonlinear Schrödinger Equation

\[
\frac{\partial A(z, T)}{\partial z} = \left( \hat{D} + \hat{N} \right) A(z, T)
\]

Remarkably simple 1+1-dimensional partial differential equation

**Dispersive operator:**

\[
\hat{D} = -\frac{\alpha}{2} - \sum_{n \geq 2} \beta_n \frac{i^{n-1}}{n!} \frac{\partial^n}{\partial T^n}
\]

- absorption
- dispersion

**Nonlinear operator:**

\[
\hat{N} A(z, T) = i\gamma \left( 1 + i\tau_{\text{shock}} \frac{\partial}{\partial T} \right) \times \left[ A(z, T) \int_{-\infty}^{\infty} R(t') |A(z, T - t')|^2 \, dt' \right]
\]

- dispersion of nonlinearity
- SPM, FWM, Raman

**Material response function:**

\[
R(t) = (1 - f_R) \delta(t) + f_R h_R(t)
\]

- instantaneous (SPM, FWM)
- delayed (Raman)

- fractional contribution of delayed response

**Nonlinear parameter:**

\[
\gamma = \frac{n_2 \omega_0}{c A_{\text{eff}}(\omega_0)}
\]

- \( n_2 \sim 3 \times 10^{-20} \, \text{m}^2/\text{W} \) (silica)
- nonlinear refractive index
Self-phase modulation (SPM)

- Pulse itself changes the refractive index of the medium (Kerr effect):
  \[ n = n_0 + n_2 I(t) = n_0 + n_2 \left( P(t)/A_{eff} \right) \]

- New frequency components are created with a time correlation:
  \[ \omega(z, t) = \omega_0 - \gamma z \partial P(t)/\partial t \]

Spectral broadening
Multi-peak structure in the spectrum
“Chirped” pulse
Soliton dynamics

Balance of SPM and anomalous dispersion

- Fundamental soliton: invariant upon propagation (except constant nonlinear phase shift)

Temporal Evolution

Spectral Evolution

Requirements:

\[ A(0, T) = \sqrt{P_0 \text{sech}(T/T_0)}; \quad N = \sqrt{\frac{\gamma P_0 T_0^2}{|\beta_2|}} = 1 \]

Solution:

\[ A(z, T) = \sqrt{P_0 \text{sech}(T/T_0)}e^{ik_{solv}z}; \quad k_{solv} = \gamma P_0/2; \]
Soliton dynamics

Higher order solitons

• Higher order solitons are periodic upon propagation:

\[ A(z + z_{\text{sol}}, T') = A(z, T') \]

Requirements:

\[ A(0, T) = \sqrt{P_0} \text{sech}(T/T_0); \quad N = \sqrt{\frac{\gamma P_0 T_0^2}{|\beta_2|}} = 2, 3, 4 \ldots \text{ quantized!} \quad z_{\text{sol}} = \frac{\pi T_0^2}{2 |\beta_2|} \]

\[
\dot{A} = -i\beta_2 \frac{\partial^2}{\partial T^2} A \\
\beta_2 < 0 \\
\dot{N} = i\gamma |A|^2
\]
Stimulated Raman scattering

- Quantum mechanical picture: photon loses energy to phonons excited in the material
- Classical picture: amplification of “Stokes” wave red-shifted from the pump
Stimulated Raman scattering

- Quantum mechanical picture: photon loses energy to phonons excited in the material
- Classical picture: amplification of “Stokes” wave red-shifted from the pump

- GNLSE includes material response:
  \[ R(t) = (1 - f_R)\delta(t) + f_R h_R(t) \]

- use analytical approximation for \( h_R(t) \) developed in literature
  \[ \rightarrow \text{e.g. Lin & Agrawal, Opt. Lett. 31, 3086 (2006)} \]

- alternative: use measured Raman spectrum \( g_R(\omega) \propto \text{Im} \left( \tilde{h}_R(\omega) \right) \)
  and Kramers-Kronig to determine real part
Soliton self-frequency shift and dispersive waves

Perturbations of N = 1 solitons

Ideal soliton propagation is disturbed by presence of Raman scattering and higher order dispersion:

Soliton self-frequency shift:
- continuous spectral red-shift
- Soliton slows down due to lower group velocity at longer wavelengths

Higher order dispersion:
- soliton sheds energy to a dispersive wave in the normal dispersion regime
- position determined by phase matching condition
Soliton fission

Perturbations of higher order solitons

- Higher order soliton propagation disturbed by Raman scattering and higher order dispersion
- Break up into fundamental solitons (here: $N = 3$)
- Break up at the point of strongest temporal compression
- Pulses separate in time and spectrum due to Raman scattering

Full GNLSE
Modulational instability

- Occurs mainly in anomalous dispersion regime
- 2 pump photons annihilated, create 1 photon in each side band
- Side band position: energy / momentum conservation \(\text{\Rightarrow} \) dispersion!
- If unseeded: shot noise amplification!

\[ \text{Full GNLSE} \]
Modulational instability

- Occurs mainly in anomalous dispersion regime
- 2 pump photons annihilated, create 1 photon in each side band
- Side band position: energy / momentum conservation $\rightarrow$ dispersion!
- If unseeded: shot noise amplification!

\[ u^b \]

Full GNLSE

- Increasing peak power
- 5 ps; 10 kW
- “sea of solitons” drives spectral broadening
Dispersion engineering for SC generation

Conventional vs. ANDi design

**Conventional design**

- both normal and anomalous dispersion regions
- pumping in anomalous dispersion close to zero dispersion wavelength (ZDW)
- designed to maximize spectral bandwidth
- soliton dynamics and phase-matched 4-wave mixing play dominant role

**All-normal dispersion (ANDi) design**

- **normal dispersion** at all wavelengths
- pumping close to the minimum dispersion wavelength (MDW)
- designed for low-noise performance
- soliton dynamics and phase-matched 4-wave mixing completely suppressed
- SPM and “optical wave-breaking” play dominant role
“Conventional” supercontinuum generation

Nonlinear dynamics

Pump pulse: soliton number $N \approx 6$
Analysis of simulation results

Time-frequency analysis

\[ \sum_g^A(\omega, \tau) = \left| \int_{-\infty}^{\infty} A(t)g(t-\tau)\exp(i\omega t)dt \right|^2 \]

simulated field \quad gate pulse

(e.g. input pulse for your simulation)
Full dynamics of continuum generation

Anomalous dispersion pumping

- low required peak power
- pulse break-up
- complex profiles, fine structure
- can be affected by quantum noise (modulation instability)
Full dynamics of continuum generation

All-normal dispersion supercontinuum

- no pulse-breakup
- minimum fine structure
- Unaffected by noise (up to ~1 ps pump pulses)
Dispersion engineering for SC generation

Conventional vs. ANDi SC (Femtosecond pumping)

**Conventional supercontinuum**

Focus: spectral bandwidth
- low pump power, very broad spectra
- highly structured and complex spectral profiles
- pump pulse breaks up into multiple solitons
- temporal and spectral interference effects
- susceptible to noisy pulse-to-pulse fluctuations

**ANDi supercontinuum**

Focus: ultrafast and low-noise applications
- single ultrashort pulse maintained
- smooth, flat spectra, steep edges
- excellent pulse-to-pulse stability
- higher pump peak power required to obtain bandwidth comparable to conventional SC
Numerical solution of the GNLSE

\[
\frac{\partial A}{\partial z} + \frac{\alpha}{2} A - \sum_{k \geq 2} \frac{i^{k+1}}{k!} \beta_k \frac{\partial^k A}{\partial T^k} = i \gamma \left(1 + i \tau_{\text{shock}} \frac{\partial}{\partial T}\right) \times \left( A(z, T) \int_{-\infty}^{\infty} R(T') |A(z, T - T')|^2 \,dT'\right)
\]

- Numerics: continuous problem is solved approximately on a discrete grid of points
- Idea:
  - represent input field on a discrete temporal grid of size \( n_t \) with resolution \( \Delta T \)
  - Fourier relations then define a frequency grid with resolution \( \Delta \nu = 1/(n_t \Delta T) \)
  - propagate stepwise through \( z \) (fiber length)
- Time-domain formulation contains a few difficulties:
  - Temporal derivatives in dispersive operator and shock term can only be approximated in discrete case \( \rightarrow \) errors
  - Convolution integral difficult to compute

\( \rightarrow \) Solution: transfer into frequency domain!
Advantages of frequency domain formulation

- Time derivatives vanish:

\[ \mathcal{F} \left( \frac{\partial}{\partial T} \right) = -i(\omega - \omega_0) \]

\( \Rightarrow \) dispersive and nonlinear operators can be applied in approximation-free manner

\( \Rightarrow \) frequency domain formulation is fundamentally more accurate

- Convolution integral vanishes:

\[ \mathcal{F} \left( \int_{-\infty}^{\infty} A(\tau) B(t - \tau) d\tau \right) = \tilde{A}(\omega) \tilde{B}(\omega) \]

Explicitly:

\[ \int_{-\infty}^{\infty} R(T') |A(z, T - T')|^2 dT' = \int_{-\infty}^{\infty} [(1 - f_R)\delta(T') + f_R h_R(T')] |A(z, T - T')|^2 dT' \]

\[ = (1 - f_R) |A(z, T)|^2 + f_R \int_{-\infty}^{\infty} h_R(T') |A(z, T - T')|^2 dT' \]

Kerr effect \quad \text{stimulated Raman scattering}

\[ \mathcal{F} \left( \int_{-\infty}^{\infty} h_R(T') |A(z, T - T')|^2 dT' \right) = \mathcal{F}(h_R(T')) \mathcal{F}(|A(z, T)|^2) \]
Frequency domain version of the GNLSE

\[
\frac{\partial \tilde{A}(z, \omega)}{\partial z} = \left( \hat{D}(\omega) + \tilde{\mathcal{N}}(z, \omega) \right) \tilde{A}(z, \omega)
\]

\[
\hat{D}(\omega) = -\frac{\alpha(\omega)}{2} + i(\beta(\omega) - \beta_1(\omega - \omega_0) - \beta(\omega_0))
\]

\[
\tilde{\mathcal{N}}(z, \omega) \tilde{A}(z, \omega) = i\gamma \left( 1 + \frac{\omega - \omega_0}{\omega_0} \right)
\]

\[
\times \mathcal{F} \left\{ (1 - f_R)|A(z, T)|^2 A(z, T) + f_R A(z, T) \mathcal{F}^{-1} \left( \mathcal{F}(h_R(T)) \mathcal{F}(|A(z, T)|^2) \right) \right\}
\]

Validity: does the envelope approximation break down for very short pulses?

\(\Rightarrow\) No, still valid even for single cycle and sub-cycle pulses!

\(\Rightarrow\) e.g. Genty et al., Opt. Express 15, 5382 (2007)
Errors caused by approximate treatment of derivatives in time-domain

"Correct" frequency domain simulation

Rieznik et al., Photonics Journal 4, 556 (2012)
Numerical solution of the GNLSE

Split-step Fourier method

\[
\frac{\partial \tilde{A}(z, \omega)}{\partial z} = \left( \hat{D}(\omega) + \hat{N}(z, \omega) \right) \tilde{A}(z, \omega)
\]

Idea: dispersion and nonlinear operator act independently over small propagation step \( h \)

\[
\tilde{A}(z = 0, \omega) \quad \text{dispersion only} \quad \text{nonlinearity only} \quad \tilde{A}(z, \omega)
\]

\[
\tilde{A}(z + h, \omega) \approx \exp \left( \frac{h}{2} \hat{D} \right) \exp \left( \int_{z}^{z+h} \hat{N}(z') \, dz' \right) \exp \left( \frac{h}{2} \hat{D} \right) \tilde{A}(z, \omega)
\]

Error: in reality, dispersion and nonlinearity act together

\[
\tilde{A}_{\text{calc}}(z + h, \omega) = \tilde{A}_{\text{true}}(z + h, \omega) + \mathcal{O}(h^3)
\]
Numerical solution of the GNLSE

Runge-Kutta in the Interaction Picture (RK4IP)

\[ \frac{\partial \tilde{A}(z, \omega)}{\partial z} = \left( \hat{D}(\omega) + \hat{N}(z, \omega) \right) \tilde{A}(z, \omega) \]

**Idea:** intelligent combination of the split-step Fourier method and an efficient integration of the nonlinear step using a Runge-Kutta algorithm

*Explicit algorithm to propagate* \( \tilde{A}(z, \omega) \rightarrow \tilde{A}(z + h, \omega) \)

\[ \Rightarrow \text{see Hult, J. Lightwave Technol. 25, 3770 (2007)} \]

Error of RK4IP method:

\[ \tilde{A}_{\text{calc}}(z + h, \omega) = \tilde{A}_{\text{true}}(z + h, \omega) + \mathcal{O}(h^5) \]
Adaptive step size algorithms

Efficient and fast calculations

Concept:
- Estimate current error during calculation
- Increase or decrease step size in order to maintain a given level of accuracy
  ➔ Makes the calculation significantly faster
  ➔ Avoids manual search for an appropriate constant step size

Examples of methods:
- Photon number conservation
  Heidt, J. Lightwave Technol. 27, 3984 (2009)
- Step size doubling
Implementation details

Determining grid sizes

2 constraints:

- **Sampling frequency > maximum frequency of the field (Nyquist)**

\[ \lambda_{\text{min}} = \frac{c}{2\Delta t + \frac{\omega_0}{2\pi}} = \frac{1}{2c\Delta t + \frac{1}{\lambda_0}} \quad \lambda_{\text{min}} \sim 500 \text{ nm} \rightarrow \Delta t < 2 fs \]

- **Width of the grid > maximum time delay of the field**

Maximum delay \(\sim 5 \text{ ps} \rightarrow \) Time window \(T > 10 \text{ ps} \)

Number of grid points \(n_p = \frac{T}{\Delta t} = 5000 \) (set \(n_p = 2^{13}\))

To avoid negative frequencies: \(\Delta t > \frac{\lambda_0}{(2c)} \rightarrow \Delta t > 1.41 \text{ fs}\)

**Be aware of wrap around effects if your window size is too small!**
Implementation details

Extras

• **Frequency dependent nonlinear parameter** \( \gamma(\omega_0) \rightarrow \gamma(\omega) \)

\[
\tilde{C}(z, \omega) = \left[ \frac{A_{\text{eff}}(\omega)}{A_{\text{eff}}(\omega_0)} \right]^{-1/4} \tilde{A}(z, \omega)
\]

\[
\gamma(\omega) = \frac{n_2 n_0 \omega_0}{c n_{\text{eff}}(\omega) \sqrt{A_{\text{eff}}(\omega) A_{\text{eff}}(\omega_0)}}
\]

solve GNLSE as usual. Requires knowledge of \( A_{\text{eff}}(\omega) , n_{\text{eff}}(\omega) \)

• **Non-polarization maintaining fiber**
  \( \rightarrow \) 2 coupled GNLSEs, one for each principal polarization axis.
  \( \rightarrow \) implementation / solver identical to "simple" GNLSE
  \( \rightarrow \) e.g. Bravo Gonzalo et al., Sci. Rep. 8, 6579 (2018), “Methods”

• **Multimode fiber**
  \( \rightarrow \) many coupled GNLSEs
  \( \rightarrow \) gets complicated
Noise properties of SC sources

Conventional supercontinuum

Shot-to-shot fluctuations: **50 fs** pump pulse (10 kW)

Simulations including shot noise

(best case scenario excluding any technical noise)
Noise properties of SC sources

Conventional supercontinuum

Simulations including shot noise
(best case scenario excluding any technical noise)

Shot-to-shot fluctuations: **150 fs** pump pulse (10 kW)
Noise in GNLSE simulations

One photon per mode model

Shot noise can be included into the simulations by injecting one photon with random phase into each spectral simulation bin $\omega_m$:

$$\tilde{A}_{\text{oppm}}(\omega_m) = \sqrt{\hbar(n_p - 1)}dT\omega_m \exp(-i\Phi(\omega_m))$$

$\Phi(\omega_m)$ randomly sampled in interval $[0, 2\pi]$.

This oppm field is then added to the input pulse:

$$A(z = 0, T) = A_{\text{input pulse}}(T) + \mathcal{F}^{-1}\left(\tilde{A}_{\text{oppm}}(\omega)\right)$$

Noise floor important for correct simulation of noise-seeded nonlinearities:
- Modulational instability
- Raman effect
Noise properties of SC sources

Conventional supercontinuum

Quantify shot-to-shot fluctuations by first-order coherence function at zero path difference

\[ g_{12}^{(1)}(\omega) = \frac{\langle \tilde{A}_i^*(\omega)\tilde{A}_j(\omega) \rangle_{i=x,j}}{\sqrt{\langle |\tilde{A}_i(\omega)|^2 \rangle \langle |\tilde{A}_j(\omega)|^2 \rangle}} \]

- \( g = 1 \): perfect amplitude / phase stability
- \( g = 0 \): random fluctuations

20 independent simulations \( \Rightarrow \) 190 unique pairs

Increasing noise amplification
Increasing “apparent” flatness

Pump pulse duration (10 kW peak power)

- 50 fs
- 100 fs
- 150 fs
Noise properties of SC sources

ANDi supercontinuum

Quantify shot-to-shot fluctuations by first-order coherence function at zero path difference

\[ |g_{12}^{(1)}(\omega)| = \left| \frac{\langle \tilde{A}_i^*(\omega)\tilde{A}_j(\omega) \rangle_{i\neq j}}{\sqrt{\langle \tilde{A}_i(\omega) \rangle^2 \langle |\tilde{A}_j(\omega)|^2 \rangle}} \right| \]

\( g = 1 \): perfect amplitude / phase stability
\( g = 0 \): random fluctuations
20 independent simulations \( \Rightarrow \) 190 unique pairs

Pump pulse duration (50 kW peak power)

Decoherence only becomes significant at \( T_p \approx 1 \) ps (vs. \( \sim 100 \) fs in conventional SC generation)

Noise properties are sensitive to fiber design!
Conclusions

- **Nonlinear fiber optics** provides powerful tools to shape laser pulses
  - in the spectral domain
  - in the temporal domain
  - in their noise and coherence properties

- **Numerical simulations** based on the GNLSE help to
  - understand nonlinear effects and their interaction
  - design new light sources with properties tailored to specific applications

- Using tips of this webinar and mentioned resources **coding your own simulation** is not difficult!

Have fun exploring nonlinear optics!
Sources of sample code

- Book “Supercontinuum Generation in Optical Fibers”
  - contains some simple sample code suitable for solving with Matlab’s internal ODE solver
  - This code does not contain full complexity of GNLSE

- www.freeopticsproject.org
  - Complete Matlab scripts to download
  - GNLSE with RK4IP solver and adaptive step size control using photon number conservation
  - Good starting point to customize your own code
Where to get fiber parameters?

- **Commercial mode solving software**
  - e.g. COMSOL Multiphysics
  - Solves stationary Maxwell equations with the boundary conditions of the fiber geometry
  - Extracts dispersion and mode field parameters

- **Empirical models**
  - Exist for an increasing number of specialty optical fibers
  - Provide empirical fitting values to generate dispersion profiles directly from fiber design parameters
  - Excellent for quickly scanning over a large range of fiber designs
  - For hexagonal PCF structures: